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THE THEORY AND APPLICATION OF DECISION ANALYSIS

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RESEARCH PROGRAM PURPOSE

The purpose of this research program is to develop new concepts and procedures for the logical analysis of decisions involving uncertainty, complexity, and dynamic effects.

SUMMARY OF ACCOMPLISHMENTS

Since the time of the last report, we have published the summary reports on the research into the value of information that may be obtained sequentially.* Three other projects are close to completion. They should be completed during the summer and be reported formally in the fall.

The first of these concerns the economics of decision analysis, including the economic balancing of assessment and computation. The second investigates the role of mathematical modeling in decision analysis with a major goal of determining the sensitivity of recommendations to choices made in the modeling process. The third analyzes the value of retaining flexibility in setting decision variables and determines the effect on the problem of quantizing state variables.

Work in two other areas is now at an intermediate stage. One project explores the use of market mechanisms in system control. Another seeks a practical method for assessing preferences for dynamic lotteries with the aim of determining when such complex preferences are necessary.

Research projects at the beginning stage are studying the axiomatic foundations of the subject and searching for practical methods of using the information received from several experts.

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*Miller, III, Allen C., The Value of Sequential Information, Ph.D. Dissertation, December 1973, Research Report No. EES-DA-73-1, January 1974

We shall now describe the progress of research that is in the intermediate and advanced stages.

RESEARCH PROGRESS

THE ECONOMICS OF DECISION ANALYSES

Research has progressed rapidly in this area during the past six months. The results are currently being summarized in a Ph.D. dissertation which should be published in the fall.

The most important result is a very general theorem on the value of information. For a profit function which is quadratic in its state and decision variables, the value of any data depends only on the prior variance of the posterior means of the state variables. The theorem has practical applications for two reasons. First, a Taylor series expansion can be used to find an approximate quadratic value function for many problems. Second, the prior variance of the posterior mean is a measure of how much the mean of a probability density function will shift during a data generation process. The potential mean shift is a quantity which can be directly encoded.

Other research includes the extension of the basic theorem to the risk-sensitive case. The most important result in this area is that under certain conditions the risk aversion coefficient may be treated as a random variable.

A third area of research is the quantization of a decision variable. The result is that although fine quantization is superior to rough quantization, the expected loss from rough quantization is negligible.

THE VALUE OF INFORMATION GIVEN FLEXIBILITY

The well-known and useful "value of information" concept of decision analysis provides a logical technique for placing a dollar value on information. In order to be more precise, however, a statement about the value of information on some uncertain variable must be accompanied by a statement about which decision variables (those variables under the control of the decision maker) may be adjusted in response to the impending information.

Therefore, we make the following definition. Suppose that the important variables associated with a particular decision problem have been divided into uncertain state variables $(s_1,\ldots,s_k)$ and decision variables $(d_1,\ldots,d_m)$. We define the value of information on state variable s_i given flexibility on decision variable d_j as the economic benefit gained by utilizing information about the state variable i when setting decision variable j.

The main usefulness of the value of information with flexibility concept lies in the fact that it allows us to evaluate the economic influence of providing information to various parts of the decision making process. It thus supplies a value yardstick against which we can compare the costs of supplying and utilizing such information. The aspects of a decision problem which affect this value yardstick have been explored by studying decision problems with a quadratic value function $v(\underline{s},\underline{d})$ which can be thought of as a Taylor series approximation to an arbitrary continuous value function. An important parameter turns out to be the degree of interaction among the decision variables, specifically the second partial derivatives $\partial^2 v | \partial d_i \partial d_j$ that measure the degree to which a change in decision variable d_j influences the effect of a change in decision variable d_i on the value function.

One of the most common ways of reducing the costs of information usage is the application of quantization. Rather than learning the exact value of a state variable prior to setting a decision variable, the quantized report tells us only that the exact value lies in one particular subset of a mutually exclusive and exhaustive collection $S_1, \ldots S_N$. Using the concept of a value of information given flexibility we can design efficient quantizing systems. Two useful results are: 1) most of the information value can usually be obtained with three or more quantizing levels, and 2) if three or more quantizing levels are used, the effects of quantization are relatively insensitive to the quantizing scheme.

MATHEMATICAL MODELING AND DECISION ANALYSIS

In this portion of the research, we are developing a comprehensive account of the role of mathematical modeling in decision analysis so that we can deal meaningfully with the dissatisfactions that we often feel over the modeling aspects of particular analyses. We have previously reported on the development of a methodology for explicitly determining the effects results of a decision analysis of the modeling assumptions made within that analysis. More recent efforts have been directed toward providing a philosophical foundation for that methodology.

Any formal decision analysis must include an explicit statement of the future consequences of present actions; this statement takes its probabilistic form in the profit lottery $\{v|d,\mathcal{E}\}$ where v is the outcome variable and d is the decision variable. Ideally, we want the authentic profit lottery - that profit lottery that is fully worthy of our acceptance and belief given the knowledge and information available to us. It is easily shown that only one profit lottery may be authentic if we are to be consistent with the utility axioms.

Unfortunately, we humans are rather poor processors of information, therefore, rather poor assessors of probabilities. Instead of directly assessing the profit lottery, then, we derive it by modeling. Modeling is essentially the expansion of the profit lottery over a set of state variables s:

$$\{v|d,\mathcal{E}\} = \int_s \{v|d,s,\mathcal{E}\}\cdot\{s|\mathcal{E}\}$$

Philosophically, the state variables represent, along with the decision variable, the "causes" of the outcome variable. The appropriate causal relationships are embodied in the conditional probability distribution $\{v|d,s,\mathcal{E}\}$. In assessing this distribution, we must draw upon our knowledge of the "connectedness" of real-world phenomena. This knowledge, in turn, is grounded in a commonly-shared mechanistic world-view. It is with reference to this world-view that we confer belief or nonbelief to statements about the world.

Because of economic and technological limitations on a decision analysis, we cannot find the profit lottery through expansion exactly; rather, we must take the following approximations:

1. $v \mathrel{\widetilde{=}} g(d,s)$ for some $g \Rightarrow$

$$\{v|d,s,\mathcal{S}\} = \delta(v - g(d,s))$$

2. $\{s|\mathcal{S}\} = p(s)$ for some discrete function p

Then

$$\{v|d,\mathcal{S}\} = \sum_{s} \sigma(v - g(d,s)) \cdot p(s).$$

It is the discrepancy in the profit lottery caused by these two approximations that is the object of the previously developed methodology.

SYSTEM CONTROL USING MARKET MECHANISMS

The theory of a market economy as an alternative to centralized economic decision-making has its roots in the eighteenth century. Unfortunately, over its two hundred year development, the compelling theoretical arguments for market mechanisms have not been translated into an equally persuasive case for market solutions to practical system problems. One primary reason is the failure of the market to assure the efficient allocation of resources in the presence of externalities, which are often characteristic of complex and dynamic economic systems.

The past six months' research on the problem of evaluating the relative advantages of controlling a dynamic system by market mechanisms rather than centralized directives has been primarily problem formulation. What has emerged is a plan to explore these aspects in two system environments--in the steady state and in transition. The former concerns the optimal characteristics of a well-tuned market system, while the latter treats transition from the status quo to that steady state.

Fundamental considerations in this analytical research will be the formulation of objectives, the evaluation of alternatives according to the selected criteria, and the implementation of the optimal strategy.

PREFERENCES FOR DYNAMIC LOTTERIES

A lottery is called dynamic if the resolution of the associated uncertainty is a function of time. An example is a lottery in which a fair coin is tossed one year from now to determine next year's income. Decision makers are frequently faced with choices between dynamic lotteries, particularly in the area of capital budgeting.

Normatively, the calculation of the present certain equivalent of the general dynamic lottery requires specification of the decision maker's time preferences (his willingness to trade-off consumption of income between time periods) and his attitude toward tisk taking. The general problem is solvable; however, the required computation and preference encoding make the solution very unwieldy.

The object of this research project is to determine if it is always necessary to consider time and risk preferences jointly when faced with the problem of choosing between two dynamic lotteries. Hopefully, in some important cases, a rational choice can be made on simplified computation and partial specification of the decision maker's preferences.

To date this research has been directed primarily at understanding the problem and studying the properties of simplified computation procedures, such as exponential utility over the present value and discounted certain equivalents.